Modelos de Sistemas Mecánicos

$$\sum_{external} \text{forces} = Ma$$

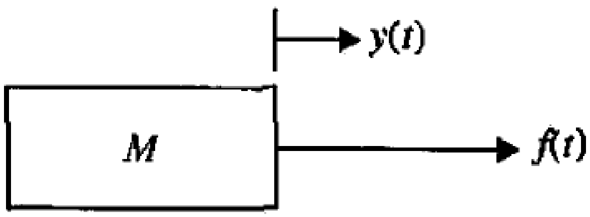


Figure 4-1 Force-mass system.

where M denotes the mass, and a is the acceleration in the direction considered. Fig. 4-1 illustrates the situation where a force is acting on a body with mass M. The force equation is written as

$$f(t) = Ma(t) = M\frac{d^2y(t)}{dt^2} = M\frac{dv(t)}{dt} \tag{4-3}$$

Modelos de Sistemas Mecánicos

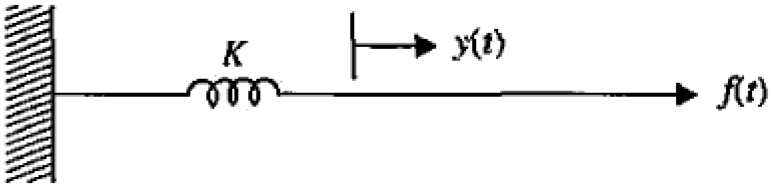


Figure 4-2 Force-spring system.

- **Linear spring.** In practice, a linear spring may be a model of an actual spring or a compliance of a cable or a belt. In general, *a spring is considered to be an element that stores potential energy.*

$$f(t) = Ky(t) \tag{4-4}$$

Modelos de Sistemas Mecánicos

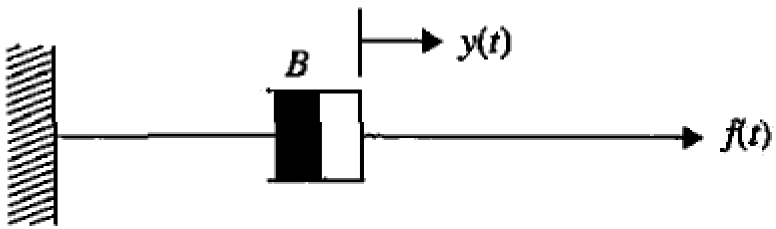


Figure 4-3 Dashpot for viscous friction.

- **Viscous friction.** *Viscous friction represents a retarding force that is a linear relationship between the applied force and velocity*. The schematic diagram element for viscous friction is often represented by a dashpot, such as that shown in Fig. 4-3. The mathematical expression of viscous friction is

$$f(t) = B\frac{dy(t)}{dt} \tag{4-6}$$

Spring Constant	*K*	N/m	lb/ft
Viscous Friction Coefficient	*B*	N/m/sec	lb/ft/sec

Modelos de Sistemas Mecánicos

Consider the mass-spring-friction system shown in Fig. 4-5(a). The linear motion concerned is in the horizontal direction. The free-body diagram of the system is shown in Fig. 4-5(b). The force equation of the system is

$$f(t) - B\frac{dy(t)}{dt} - Ky(t) = M\frac{d^2y(t)}{dt^2} \tag{4-9}$$

The last equation may be rearranged by equating the highest-order derivative term to the rest of the terms:

$$\frac{d^2y(t)}{dt^2} = -\frac{B}{M}\frac{dy(t)}{dt} - \frac{K}{M}y(t) + \frac{1}{M}f(t) \tag{4-10}$$

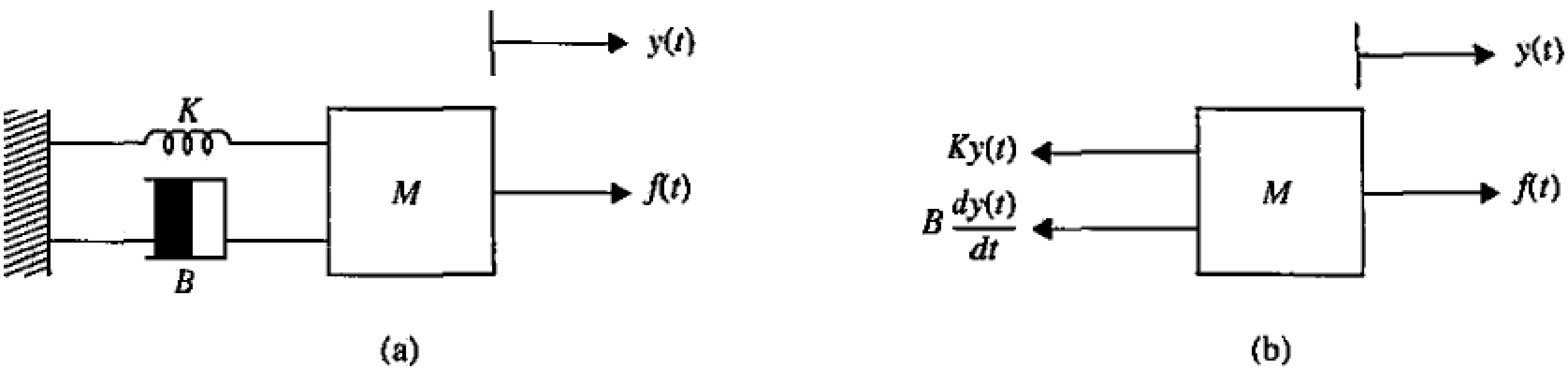


Figure 4-5 (a) Mass-spring-friction system. (b) Free-body diagram.

Modelos de Sistemas Mecánicos

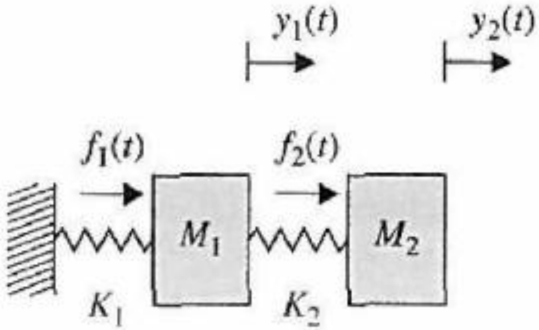


Figure 4-12 A 2-DOF spring-mass system.

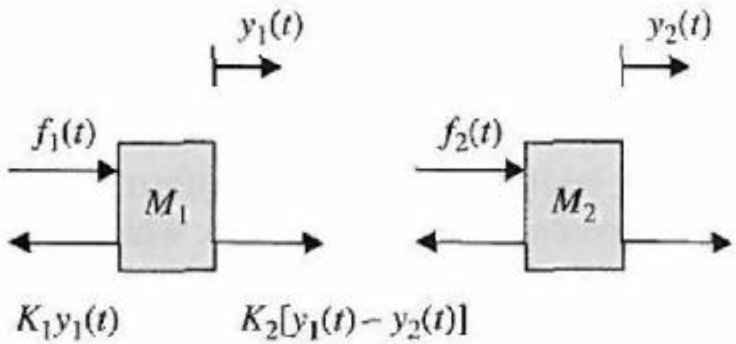


Figure 4-13 FBD of the 2-DOF spring-mass system.

$$\begin{aligned} f_1(t) - K_1 y_1 + K_2(y_1 - y_2) &= M_1 \ddot{y}_1 \\ f_2(t) - K_2(y_1 - y_2) &= M_2 \ddot{y}_2 \end{aligned} \tag{4-28}$$

Rearranging the equations into the standard input–output form, we have

$$\begin{aligned} M_1\ddot{y}_1 + (K_1 + K_2)y_1 - K_2 y_2 &= f_1(t) \\ M_2\ddot{y}_1 - K_2 y_1 + K_2 y_2 &= f_2(t) \end{aligned} \tag{4-29}$$

Movimiento Circular

The rotational motion of a body can be defined as motion about a fixed axis. The extension of Newton's law of motion for rotational motion states that the *algebraic sum of moments or torque about a fixed axis is equal to the product of the inertia and the angular acceleration about the axis*. Or

$$\sum \text{torques} = J\alpha \tag{4-33}$$

- **Inertia.** *Inertia, J, is considered a property of an element that stores the kinetic energy of rotational motion.* The inertia of a given element depends on the geometric composition about the axis of rotation and its density. For instance, the inertia of a circular disk or shaft, of radius r and mass M, about its geometric axis is given by

$$J = \frac{1}{2}Mr^2 \tag{4-34}$$

Inertia	J	kg-m^2

Movimiento Circular

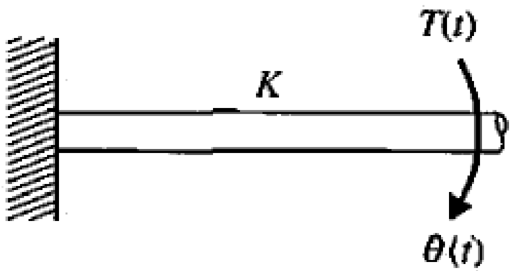


Figure 4-15 Torque torsional spring system.

- **Torsional spring.** As with the linear spring for translational motion, a **torsional spring constant K**, in torque-per-unit angular displacement, can be devised to represent the compliance of a rod or a shaft when it is subject to an applied torque. Fig. 4-15 illustrates a simple torque-spring system that can be represented by the equation

$$T(t) = K\theta(t) \tag{4-36}$$

Movimiento Circular

- **Friction for rotational motion.** The three types of friction described for translational motion can be carried over to the motion of rotation. Therefore, Eqs. (4-6), (4-7), and (4-8) can be replaced, respectively, by their counterparts:
  - **Viscous friction.**

$$T(t) = B\frac{d\theta(t)}{dt} \tag{4-38}$$

Movimiento Circular

Fig. 4-17(a) shows the diagram of a motor coupled to an inertial load through a shaft with a spring constant K. A non-rigid coupling between two mechanical components in a control system often causes torsional resonances that can be transmitted to all parts of the system. The system variables and parameters are defined as follows:

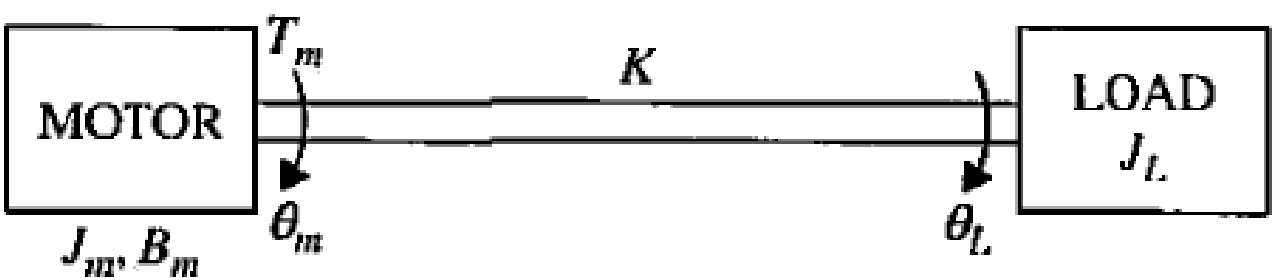
$T_m(t)$ = motor torque

B_m = motor viscous-friction coefficient

K = spring constant of the shaft

$\theta_m(t)$ = motor displacement

$\omega_m(t)$ = motor velocity



(a)

Movimiento Circular

J_m = motor inertia

$\theta_L(t)$ = load displacement

$\omega_L(t)$ = load velocity

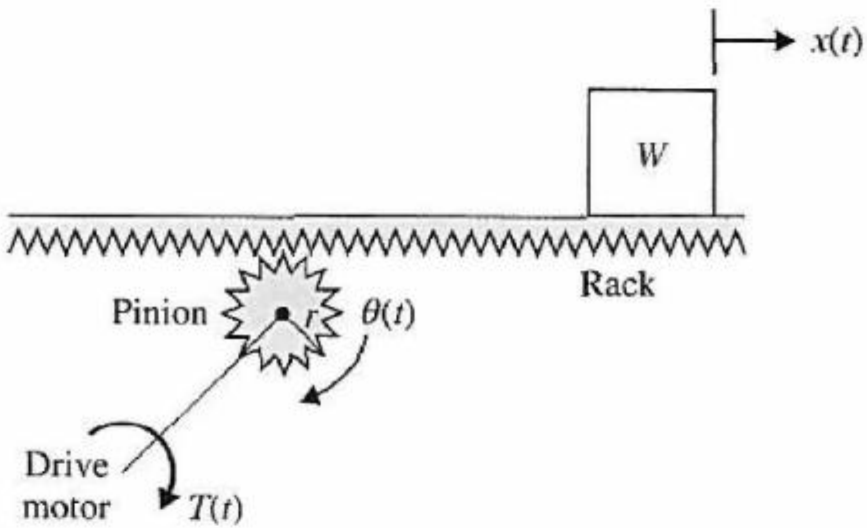
J_L = load inertia

The free-body diagrams of the system are shown in Fig. 4-17(b). The torque equations of the system are

$$\frac{d^2\theta_m(t)}{dt^2} = -\frac{B_m}{J_m}\frac{d\theta_m(t)}{dt} - \frac{K}{J_m}[\theta_m(t) - \theta_L(t)] + \frac{1}{J_m}T_m(t) \quad (4\text{-}42)$$

$$K[\theta_m(t) - \theta_L(t)] = J_L\frac{d^2\theta_L(t)}{dt^2} \quad (4\text{-}43)$$

Conversión de Movimiento



$$J = Mr^2$$

Figure 4-20 Rotary-to-linear motion control system (rack and pinion).

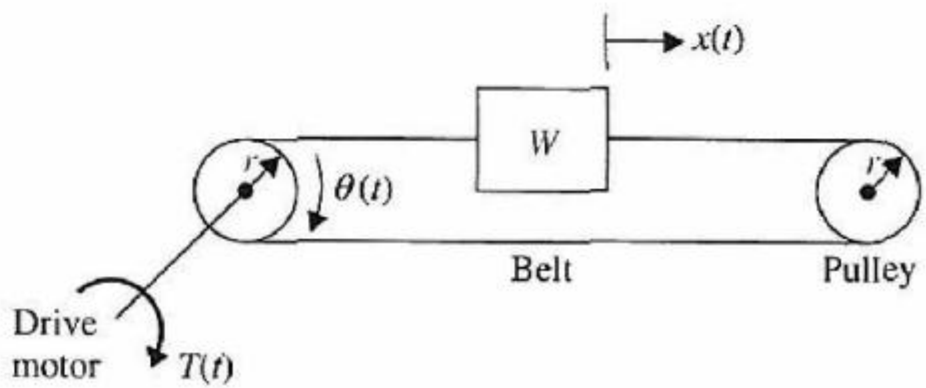


Figure 4-21 Rotary-to-linear motion control system (belt and pulley).

Engranajes (Modelo Ideal)

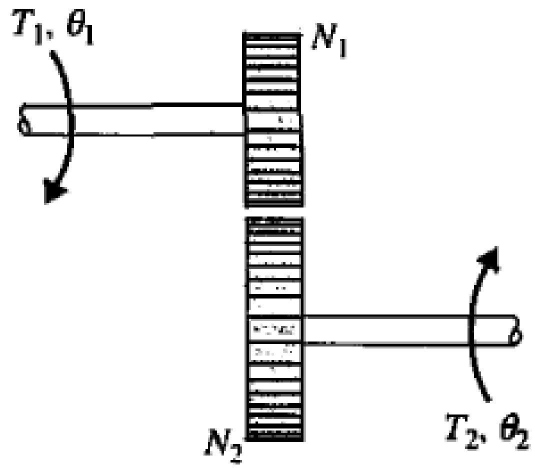
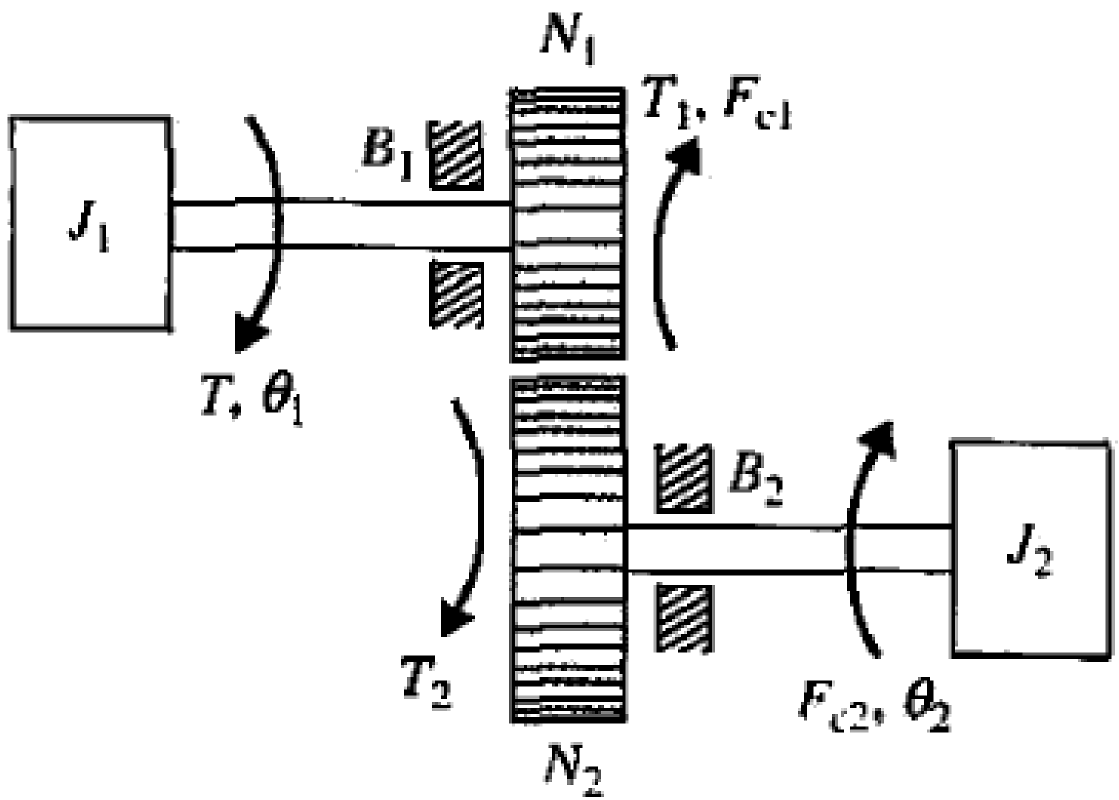


Figure 4-22 Gear train.

If the angular velocities of the two gears ω_1 and ω_2 are brought into the picture, Eqs. (4-49) through (4-51) lead to

$$\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{N_1}{N_2} = \frac{\omega_2}{\omega_1} = \frac{r_1}{r_2} \tag{4-52}$$

Engranajes (Modelo No Ideal)



Engranajes (Modelo No Ideal)

$$T(t) = J_1 \frac{d^2\theta_1(t)}{dt^2} + B_1 \frac{d\theta_1(t)}{dt} + F_{c1} \frac{\omega_1}{|\omega_1|} + T_1(t)$$

where T denotes the applied torque, T_1 and T_2 are the transmitted torque, F_{c1} and F_{c2} are the Coulomb friction coefficients, and B_1 and B_2 are the viscous friction coefficients. The torque equation for gear 2 is

$$T_2(t) = J_2 \frac{d^2\theta_2(t)}{dt^2} + B_2 \frac{d\theta_2(t)}{dt} + F_{c2} \frac{\omega_2}{|\omega_2|}$$

Engranajes (Modelo No Ideal)

Using Eq. (4-52), Eq. (4-53) is converted to

$$T_1(t) = \frac{N_1}{N_2} T_2(t) = \left(\frac{N_1}{N_2}\right)^2 J_2 \frac{d^2\theta_1(t)}{dt^2} + \left(\frac{N_1}{N_2}\right)^2 B_2 \frac{d\theta_1(t)}{dt} + \frac{N_1}{N_2} F_{c2} \frac{\omega_2}{|\omega_2|} \quad (4\text{-}55)$$

Eq. (4-55) indicates that it is possible to reflect inertia, friction, compliance, torque, speed, and displacement from one side of a gear train to the other. The following quantities are obtained when reflecting from gear 2 to gear 1:

$$\begin{aligned} &\text{Inertia}: \left(\frac{N_1}{N_2}\right)^2 J_2 \\ &\text{Viscous-friction coefficient}: \left(\frac{N_1}{N_2}\right)^2 B_2 \\ &\text{Torque}: \frac{N_1}{N_2} T_2 \\ &\text{Angular displacement}: \frac{N_1}{N_2} \theta_2 \\ &\text{Angular velocity}: \frac{N_1}{N_2} \omega_2 \end{aligned} \quad (4\text{-}56)$$

Engranajes (Modelo No Ideal)

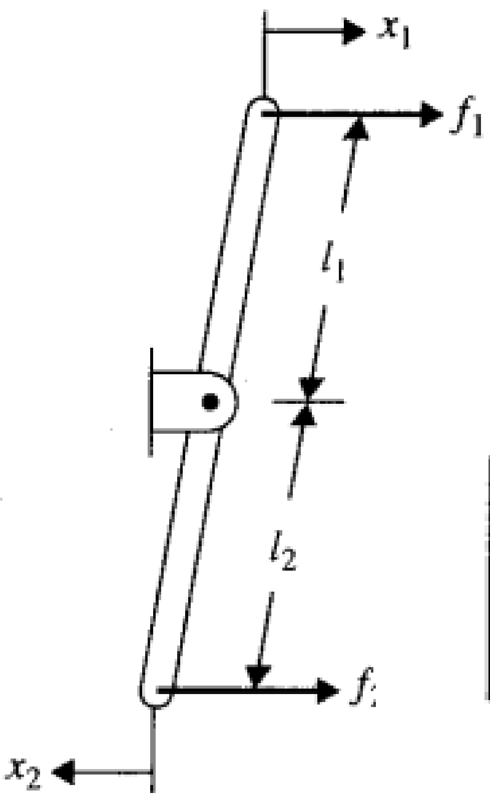
$$T(t) = J_{1e}\frac{d^2\theta_1(t)}{dt^2} + B_{1e}\frac{d\theta_1(t)}{dt}.$$

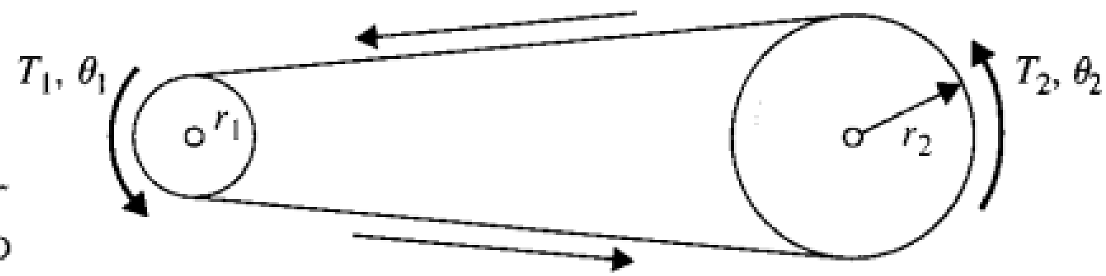
$$J_{1e} = J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2$$

$$B_{1e} = B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2$$

Correas y Palancas

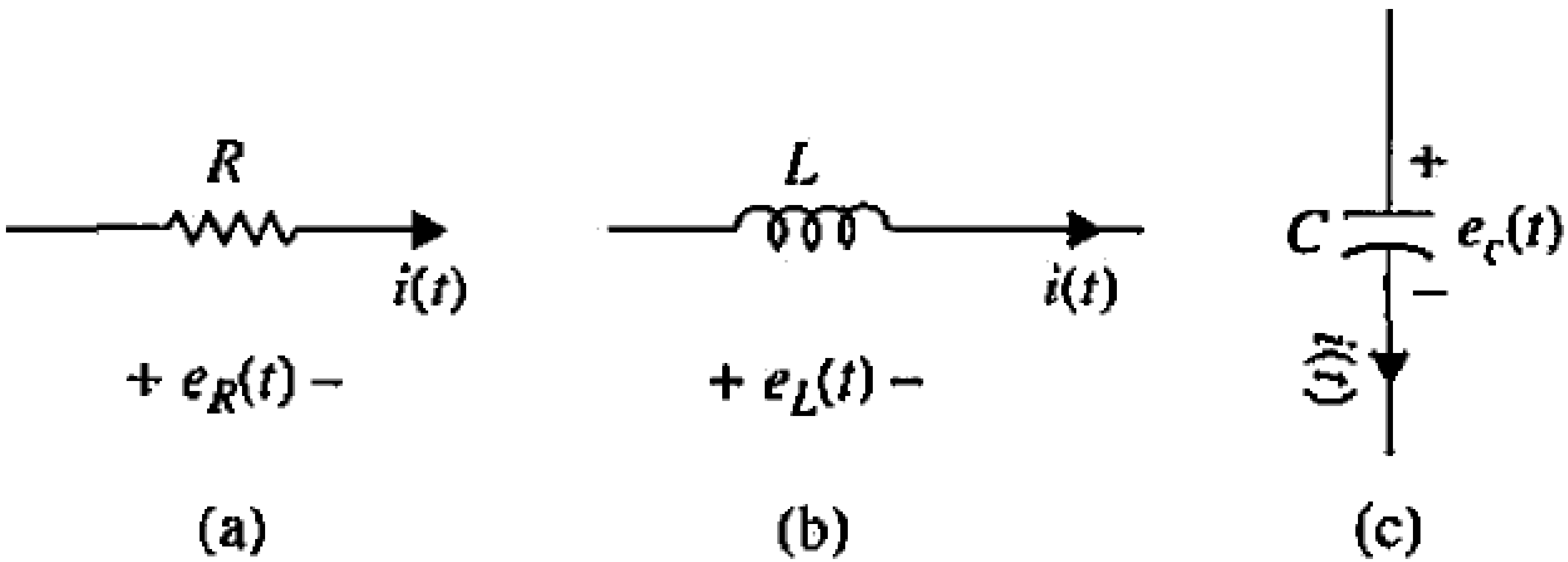
Las bandas y cadenas sirven para el mismo propósito que el tren de engranes excepto que permiten la transferencia de energía sobre una distancia mayor sin utilizar un número excesivo de engranes. La Fig. 4-14 muestra el diagrama de una cadena o banda entre dos poleas. Suponiendo que no hay deslizamiento entre la banda y la polea, es fácil ver que la





$$\frac{f_1}{f_2} = \frac{l_2}{l_1} = \frac{x_2}{x_1}$$

Sistemas Eléctricos

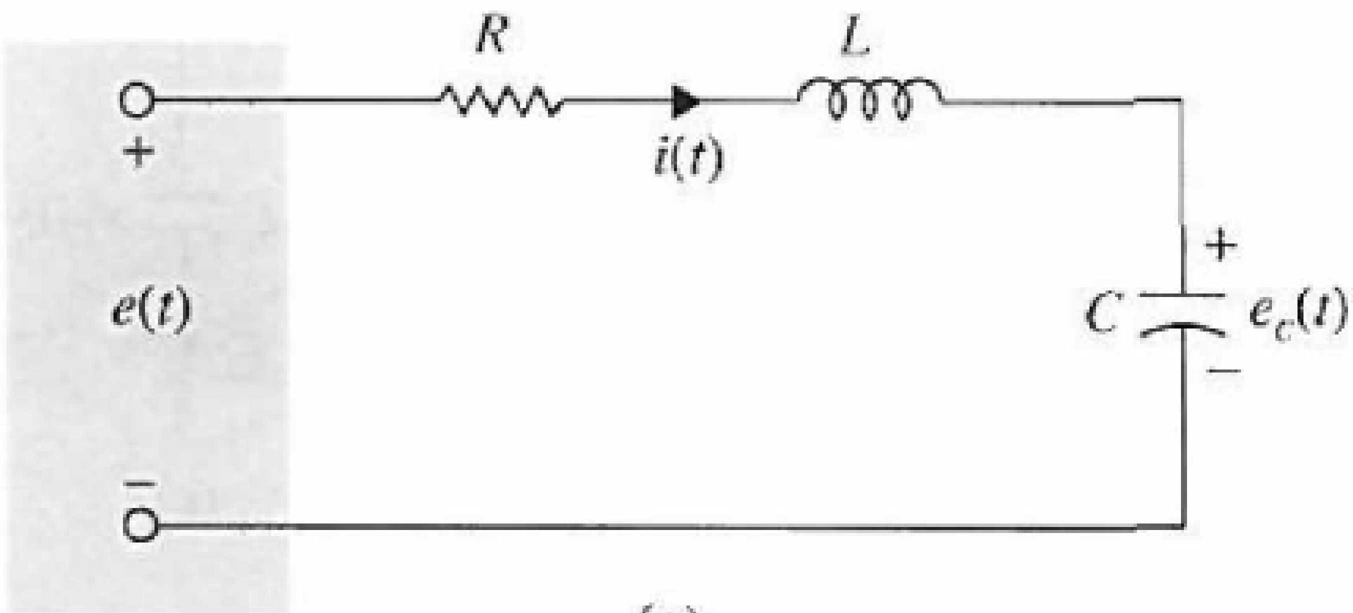


$$e_R(t) = i(t)R$$

$$e_L(t) = L\frac{di(t)}{dt}$$

$$e_c(t) = \int \frac{i(t)}{C}dt$$

Sistemas Eléctricos



(a)

Sistemas Eléctricos

where

$e_R =$ Voltage across the resistor R

$e_L =$ Voltage across the inductor L

$e_c =$ Voltage across the capacitor C

Or

$$e(t) = +e_c(t) + Ri(t) + L\frac{di(t)}{dt}$$

Using current in C:

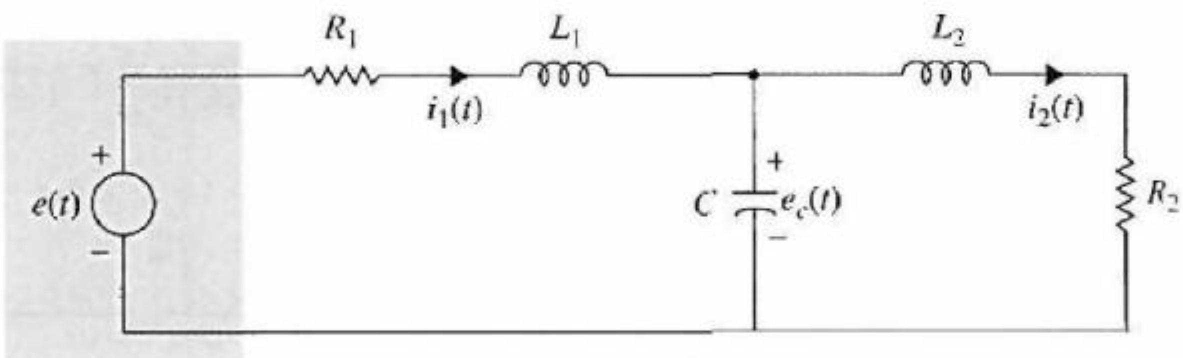
$$C\frac{de_c(t)}{dt} = i(t)$$

and taking a derivative of Eq. (4-54) with respect to time, we get the equation

$$L\frac{d^2i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{i(t)}{C} = \frac{de(t)}{dt}$$

Sistemas Eléctricos

As another example of writing the state equations of an electric network, consider the network shown in Fig. 4-28(a). According to the foregoing discussion, the voltage across the capacitor, $e_c(t)$, and the currents of the inductors, $i_1(t)$ and $i_2(t)$, are assigned as state variables, as shown in Fig. 4-28(a). The state equations of the network are obtained by writing the voltages across the



(a)

Sistemas Eléctricos

$$L_1 \frac{di_1(t)}{dt} = -R_1 i_1(t) - e_c(t) + e(t)$$

$$L_2 \frac{di_2(t)}{dt} = -R_2 i_2(t) + e_c(t)$$

$$C \frac{de_c(t)}{dt} = i_1(t) - i_2(t)$$

Sistemas Eléctricos

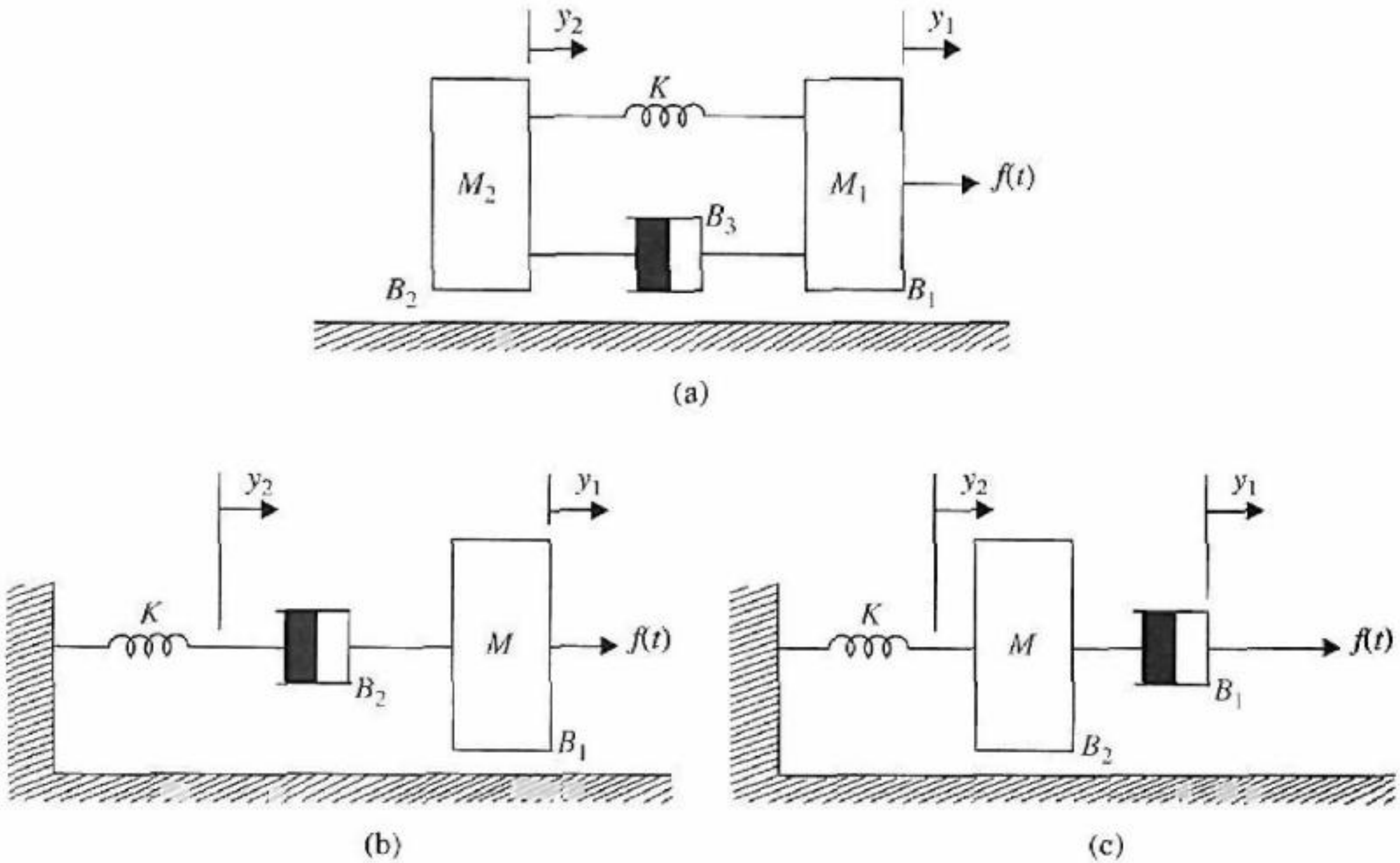
$$L_1 \frac{di_1(t)}{dt} = -R_1 i_1(t) - e_c(t) + e(t)$$

$$L_2 \frac{di_2(t)}{dt} = -R_2 i_2(t) + e_c(t)$$

$$C \frac{de_c(t)}{dt} = i_1(t) - i_2(t)$$

Ejercicios

4-4. Write the force equations of the linear translational systems shown in Fig. 4P-4.



Ejercicios

4-6. Consider a train consisting of an engine and a car, as shown in Fig. 4P-6.

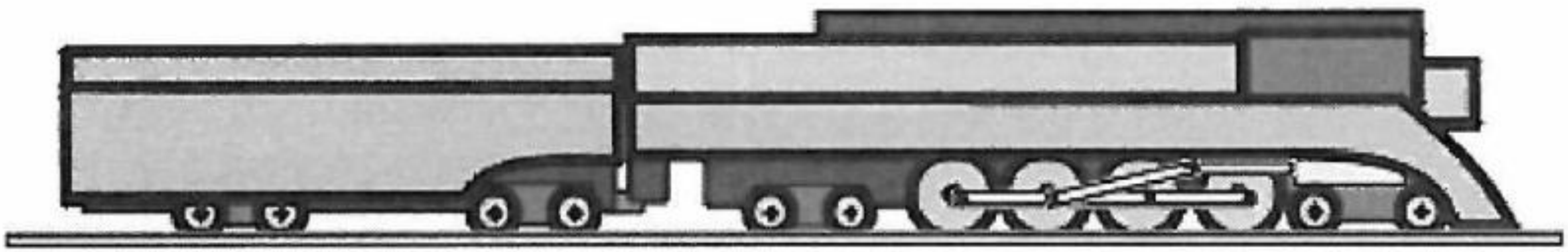


Figure 4P-6

A controller is applied to the train so that it has a smooth start and stop, along with a constant-speed ride. The mass of the engine and the car are M and m, respectively. The two are held together by a spring with the stiffness coefficient of K. F represents the force applied by the engine, and μ represents the coefficient of rolling friction. If the train only travels in one direction:

(a) Draw the free-body diagram.

(b) Find the state variables and output equations.

Ejercicios

4-7. A vehicle towing a trailer through a spring-damper coupling hitch is shown in Fig. 4P-7. The following parameters and variables are defined: M is the mass of the trailer; K_h, the spring constant of the hitch; B_h, the viscous-damping coefficient of the hitch; B_t, the viscous-friction coefficient of the trailer; $y_1(t)$, the displacement of the towing vehicle; $y_2(t)$, the displacement of the trailer; and $f(t)$, the force of the towing vehicle.

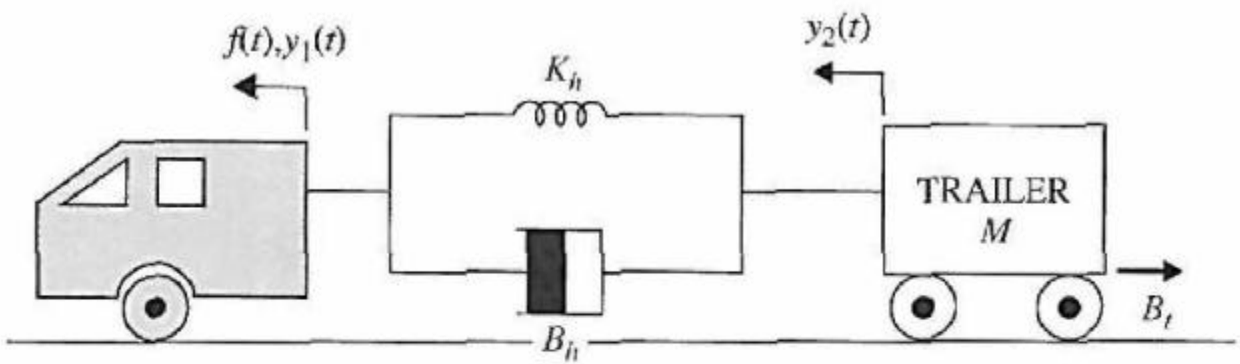
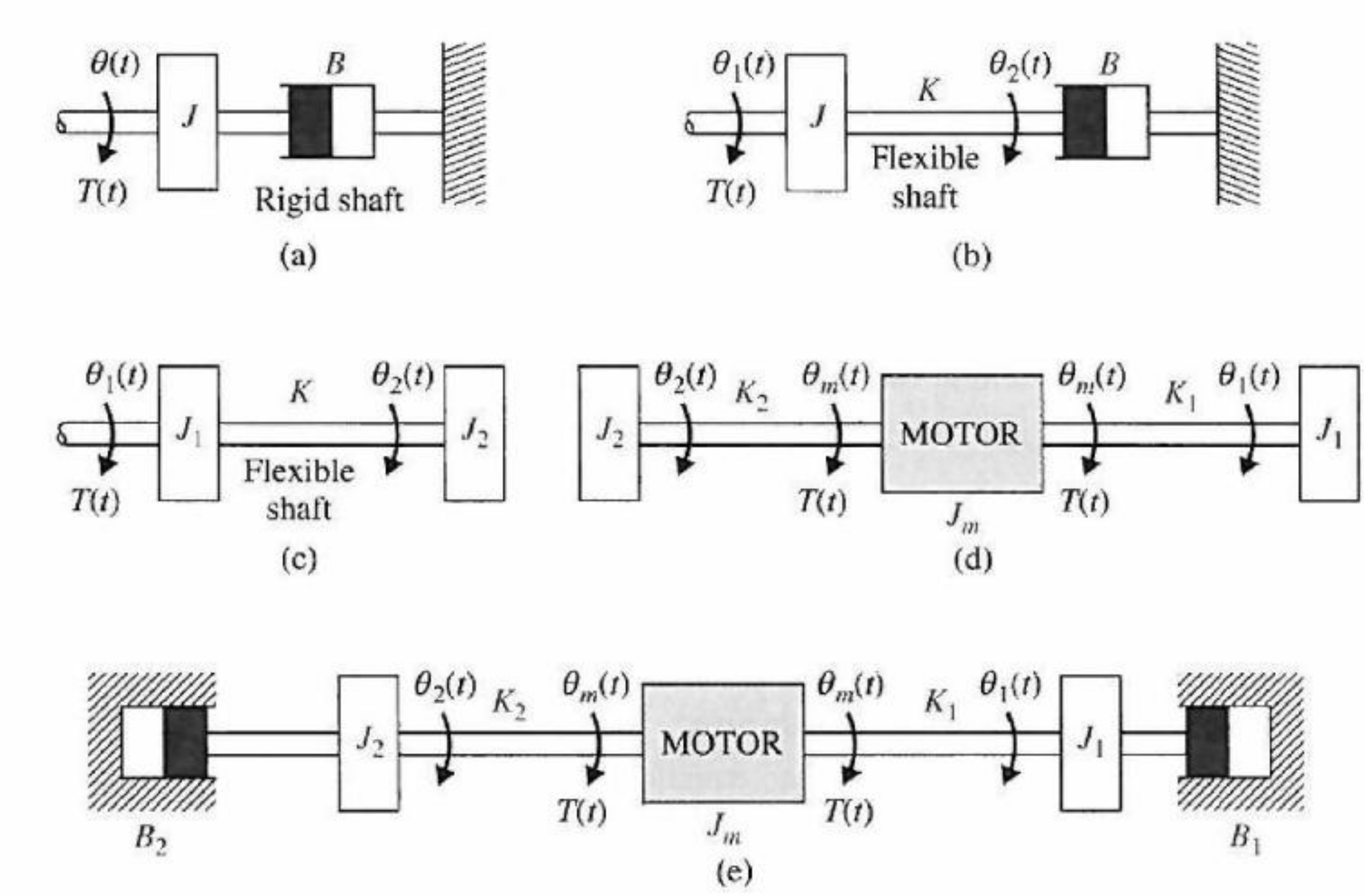
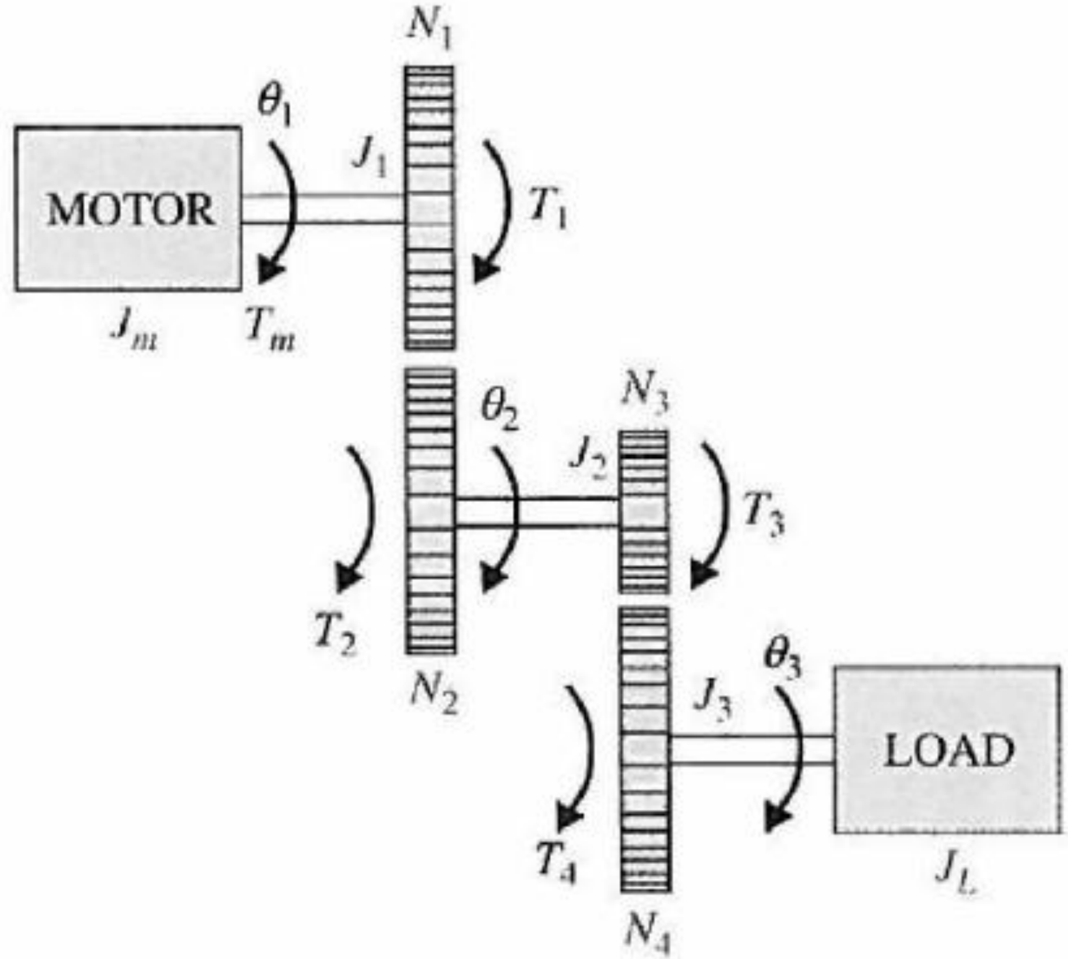


Figure 4P-7

Ejercicios

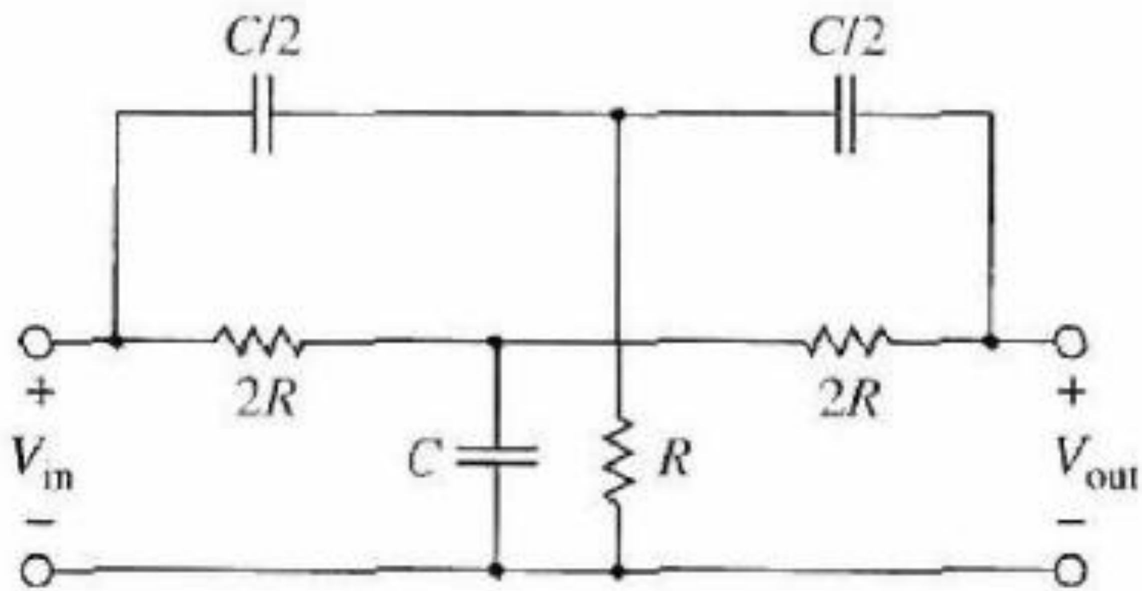

θ(t)
B
J
T(t)
Rigid shaft
(a)
θ1(t)
θ2(t)
B
K
J
Flexible shaft
T(t)
(b)
θ1(t)
K
θ2(t)
J1
J2
T(t)
Flexible shaft
(c)
J2
θ2(t)
K2
θm(t)
MOTOR
θm(t)
K1
θ1(t)
J1
T(t)
T(t)
Jm
(d)
θ2(t)
K2
θm(t)
θm(t)
K1
θ1(t)
J2
MOTOR
J1
T(t)
T(t)
Jm
B2
B1
(e)

Ejercicios

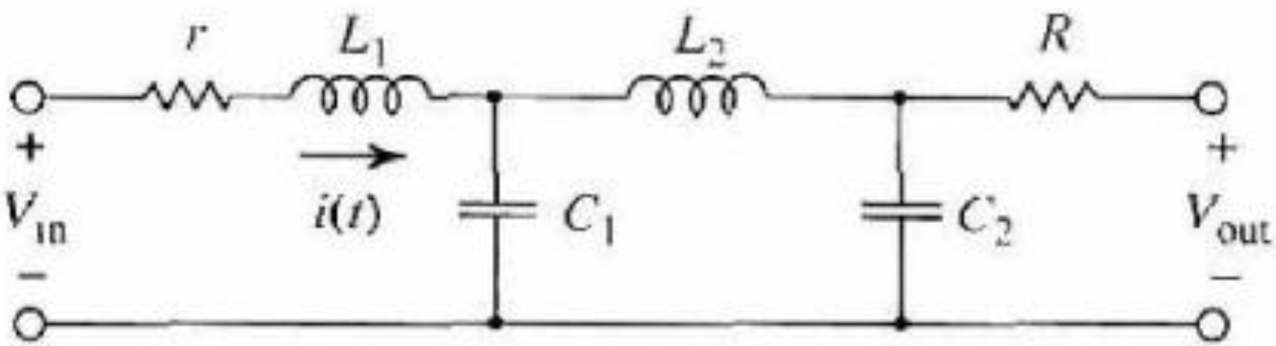


N_1
θ_1
MOTOR
J_1
T_1
J_m
T_m
N_3
θ_2
J_2
T_3
T_2
N_2
θ_3
J_3
LOAD
T_4
J_L
N_4

Ejercicios



(a)



(b)